



## PROPOSING A MATHEMATICAL SOFTWARE TOOL IN PHYSICS SECONDARY EDUCATION

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**Abstract:** MathCad<sup>®</sup> is a very popular software tool for mathematical and statistical analysis in science and engineering. Its low cost, ease of use, extensive function library, and worksheet-like user interface distinguish it among other commercial packages. Its features are also well suited to educational process. The use of natural mathematical notation and built-in measurement units are its two major advantages in teaching and learning. In this paper, its complementary use in the upper secondary physics education in Greece is explored. In order to demonstrate its application in the teaching process, a set of representative examples are presented. The main features and advantages of the software are also pointed out. The paper aims to present the benefits of the application of mathematical information technology tools in secondary physics education. In this effort, MathCad<sup>®</sup> is probably the most promising solution.

**Zusammenfassung:** MathCad<sup>®</sup> ist ein sehr beliebtes Software-Programmentwicklungssystem für mathematische und statistische Analyse bei der Wissenschaft- und Ingenieurwissenschaft, dessen niedrige Kosten, einfache Bedienungen, umfangreiche Funktionsbibliothek und den Arbeitsblatt-ähnliche Benutzeroberfläche, die es unter anderen kommerziellen Paketen. Seine Eigenheiten lassen sich sehr gut im Ausbildungsbereich und Schulwesen anpassen. Der Einsatz von einfachen Notationen und eingebauten Messeinheiten sind zwei von den wichtigsten Vorteilen von MathCad<sup>®</sup> im Bereich von Lehren und des Lernens. Im vorliegenden Artikel wird erforscht, in der Sekundarstufe II Physik in Griechenland untersucht. Es wird eine Reihe von ausgewählten Beispielen dargestellt, die seine Anwendung in der Ausbildung vorstellen. Ferner werden auch seine Haupteigenschaften und Vorteile aufgezeigt. Das Ziel dieses Artikels ist, die Vorteile der Anwendung und Benutzung von mathematischen Werkzeugen der Informationstechnologie in der sekundären Ausbildung Didaktik. Zu diesem Zweck, scheint MathCad<sup>®</sup> die äußerst angemessene Lösung zu sein.

**Keywords:** physics education, information technology, MathCad<sup>®</sup>, educational tool, secondary education

### 1. Introduction

Nowadays, innovative approaches to teaching science and engineering have appeared using information technologies (IT) tools for efficient education in terms of time and cost. Moreover, application of these new educational methodologies prepare students for more productive results, accelerate their progress and reemphasize their understanding of theories for the purpose of quality education (Li – Khan, 2005). Use of computers and software IT tools in classrooms and laboratories, provide much more effective and efficient environments in teaching and learning, making mathematical-related science and engineering problems easier to understand.

The advantages of using simulation software in conjunction with classroom teaching are well known. It is generally acceptable that the use of interactive teaching tools, which provide instant feedback to the student's inputs, improve and accelerate the learning process. The use of simulation and IT tools in graduate courses is not a new concept (Kezunovic et al., 2004). However, the traditional teaching methodology used in secondary education is based mainly on oral speech and use of blackboard. This situation is common in many countries, including Greece. However, in the last years, significant effort has been made to improve the Greek Secondary Education System in this aspect. The recent strategy of the Greek Ministry of Education (<http://www.ypepth.gr/>) to employ educational software in the

teaching process in the secondary education has motivated significant efforts. In the last years, more than a hundred software programs have been developed for this purpose (Andreadis–Papadimitriou et al., 2007)

Educational software is grouped into two categories, the freeware software that is freely available over the Web and the commercial educational packages; the last are usually being scheduled in relation to the program of each country's education system (Mamoukaris et al, 2000). Despite their low cost, these approaches are not always efficient. In any case, the products' well-known strengths and weakness should be carefully considered from both the teacher and the students and requisite a good teaching technique that should benefit both parties, (Fuhrman, 2007). Among others, factors that need to be taken into account are the students' background and experiences and the nature of the subject (Ciascai – Marchis, 2008).

This paper is concerned with a discussion of a software package to be used in secondary physics education. The intent is to provide students with a tool to help them solve their homework and understand the subject matter better. Students who have limited or no background in computer and simulations attain the courses; therefore, the software must be user friendly and easy to learn. It is also desirable to be reasonably fast and accurate. Surely, a properly chosen commercial package is more possible to fulfill these criteria.

Several popular commercial mathematical packages such as MATLAB<sup>®</sup> (Mathworks, 2001; Gilat, 2008), Mathematica<sup>®</sup> (Wolfram, 2003; Tam 2008) and MathCad<sup>®</sup> (Mathsoft, 2001; Maxfield, 2006; Pritchard, 2007) can be used in the teaching process. These are very powerful tools and find numerous applications in science, technology, and education (a detailed comparison of these tools is out of the scope of this paper; comparative studies can be found in published literature, see Domnisoru (2005) for example). However, the ease of use, the screen appearance (equations and numerical values are presented in an easy to read fashion), and the integration of measurement units are some of the major advantages of MathCad<sup>®</sup> compared to the other two programs. The author of this paper also believes that the skills required by the users of MATLAB<sup>®</sup> or Mathematica<sup>®</sup> restrict their use in secondary education.

The rest of the paper is organized as follows: Section 2 provides a short description of the structure of the Upper Secondary Education System in Greece and the secondary physics education. In Section 3, an overview of the pedagogical proposal is given. The key features and major advantages of the complementary use of MathCad<sup>®</sup> in secondary education are presented in Section 4. In Section 5, a discussion of related examples taken from the physics textbooks of the upper secondary education in Greece offers powerful insights. Finally, Section 6 concludes the paper.

## 2. Upper secondary education in Greece: General characteristics and physics education

Secondary education in Greece consists of the Compulsory and the Post–compulsory Secondary Education. The first is provided by Gymnasium for all children 12-15 years old and constitutes the lower level of secondary education, while Unified Lyceum and Technical Vocational Schools (TEE) provide the second (<http://www.ekep.gr/>).

The Unified Lyceum (ISCED level 3) consists of three classes and covers the first three years of post–compulsory education for students greater than 15 years old. It is focused on general education and aims to the development of the student's all–round skills and abilities. Class 1 is common for all students. During Classes 2 and 3, students choose one of the provided orientation lessons, theoretical, practical or technological, but also attend core class. After their graduation, they may seek admission to the Universities or the Technological Educational Institutes or attend Vocational Training Institutes to obtain a training certificate. Studies at the Technical Vocational Schools last up to three years and provide both general education and vocational skills to prepare students for the labor market. They consist of two independent cycles covering specific fields and specializations. After their graduation, students may also seek admission to Higher Education.

Physics education plays an important role in secondary studies. Some of the issues covered are mechanics, electromagnetics, thermodynamics, electrical circuits, waves, fluid mechanics, atomic and

nuclear physics, relativity, quantum theory, and optoelectronics. Teaching consists of both lectures and experimental courses and aims to provide students with a general framework in the basic principles of classic and modern physics and its applications in science and technology. For additional information, visit the website of the Greek Network of Schools – P.I. (<http://www.pi-schools.gr/>).

### 3. Using Mathcad® in secondary physics education: Pedagogical concepts

Vallim (Vallim et al., 2006) has abstracted three basic concepts upon which a modern pedagogical proposal should be based on:

- Learning is based on previous knowledge and experiences (Piaget, 1975).
- Learning is both a personal and a social process (Vygotsky, 1978).
- Context helps motivate learning, (Papert, 1993).

One point arises as we consider the consistency between these concepts and the use of MathCad® as a teaching tool. The students' previous experiences and educational background play an important role in their learning. Basic computer skills are prerequisite to using an IT software tool. Secondary students have acquired these skills either by teaching or through their personal experience. In any case, previous modules in which basic computer skills are taught are important. Obviously, computer simulations should follow the style of lectures. Students should be encouraged to complete the theoretical study prior to the simulations.

Personal effort is an essential component of learning. However, social interaction allows students to act as mediators of knowledge for each other. Students working in a group can solve problems that they would not be able to solve alone. At the same time, they share knowledge and experience enjoyment. Moreover, their critical and reflective thinking is encouraged and their creativity and interest increase. The spirit of teamwork and the associated concepts of task partitioning, time and budget allocation, and load distribution are further important outcomes (Halalu – Elwakil, 2004). However, this is also a risk because of differences in personality, interest, and abilities, which can create conflicts that must be managed from the teacher. The idea of working in groups of two is common and gives good results (Roberts, 2005). Use of IT tools surely motivates students' collaboration.

Application of IT tools in education improves and accelerates the learning process (Gorman et al., 2005). As long, one of the primary educational aims is to make students think independently which involves positive experiences gained in a course (Szállassy, 2008), the use of IT tools such as MathCad® fulfills this goal. Modeling and simulation were always useful in teaching physics, especially when physical interaction needed to be explained in a comprehensive way and appropriate and user-friendly computational tools were available. In that way, students were able to study processes that were not easily accessible because of their complexity and difficulty. The author believes that MathCad® has a great potential to help secondary students better understand a physics course.

### 4. Using Mathcad® in secondary physics education: Features and advantages

MathCad® (<http://www.ptc.com/>) is a popular engineering calculation software that offers significant personal and process productivity advantages for product development and engineering design projects and finds many applications in science and engineering (Delgado – Thursby, 1999; Mallick – Crawford, 2005; Esa et al., 2006; Constantinescu et al. 2007), for example. By integrating text, mathematical expressions, results, and graphs into a single environment throughout the document, it provides a solution that simultaneously solves and documents calculations, provides automatic units management, and expresses all calculations in standard math notation. Its worksheet-like user interface accepts and displays natural mathematical notation and documents the calculations and results in worksheets, which can be saved or converted to several formats. Its low cost and ease of use and the existence of extensive math libraries, interactive e-books, and extension packs have distinguished it among other commercial products.

Among its features, two are the most important in the teaching process (Domnisoru, 2005). The screen appearance of the equations matches as closely as possible to the traditional form of the equations and helps students to become familiar with the basic concepts of physics. Users of MathCad<sup>®</sup> write the equations in their natural form, which help them to avoid errors. Its second important feature, is the use of built-in measurement units; as a result, the students are completely free from keeping track of the units used. Indicating the units for each data input, and simply asking for results, transformations are automatically performed. For example, consider that the voltage drop at an ohmic resistance  $R = 47\Omega$  is  $v = 1kV$ . In order to calculate the current flow, we write in the computer screen the following code:

```
R:=47Ω [enter]
```

```
v:=1kV [enter]
```

```
I:= $\frac{v}{R}$  [enter]
```

```
I= [enter]
```

which gives  $I = 21.28A$ . Notice that not only the units transformation Volt/Ohm=Ampere takes place automatically, but also the multiplying factor 1k is considered.

## 5. Representative examples and discussions

In order to exhibit the characteristic features of the package and demonstrate its capability to meet the requirements as a complementary tool in the secondary physics education, three representative examples from the Greek physics textbooks of Unified Lyceum's Classes 2 and 3 (Ioannou et al, 2000; Ioannou et al, 2001) follow. In these examples, the application of the ideal gas equation of state (Müller, 2007), the calculation of the force on infinite length wire with current flowing (Neff, 1991) and the propagation of elastic waves (Knobel, 2000), are respectively studied.

First example (Ioannou et al, 2000, pp. 13-14): Ideal gas equation of state.

Consider 0.2mol  $H_2$  in temperature 300K and pressure 2atm (state A). The gas is heated to 400K at a standard pressure (state B). Afterwards, the pressure is decreased to 1.5atm during in an isothermal process (state C). Then, temperature is isometrically reduced to 300K (state D). Finally, the gas returns to its initial state during an isothermal process. Calculate the volume, pressure, and temperature at the various states. The molar gas constant is  $R = 0.082L \cdot atm \cdot mol^{-1} \cdot K^{-1}$ .

Input data are introduced as:

```
R := 0.082  $\frac{l \cdot atm}{mol \cdot k}$  [enter]
```

```
n:=0.2mol [enter]
```

```
pA:=2atm [enter]
```

```
pC:=1.5atm [enter]
```

```
TA:=300K [enter]
```

```
TB:=400K [enter]
```

Problem solution consists of simply typing the basic formulas derived from the ideal gas equation of state and the kind of process (isobaric, isothermal, isometric). It is:

$$V_A := \frac{nRT_A}{p_A} \text{ [enter]}$$

$$p_B := p_A \text{ [enter]}$$

$$V_B := \frac{T_B}{T_A} V_A \text{ [enter]}$$

$$T_C := T_B \text{ [enter]}$$

$$V_C := \frac{p_B}{p_C} V_B \text{ [enter]}$$

$$V_D := V_C \text{ [enter]}$$

$$T_D := T_A \text{ [enter]}$$

$$p_D := \frac{T_D}{T_C} p_C \text{ [enter]}$$

Calculation of the results is straightforward. For example, the gas volume at state C can be found by writing  $V_C =$  [enter] which gives  $V_C = 4.37$ l; writing  $p_D =$  [enter] we get that  $p_D = 1.14 \times 10^5$  Pa, the gas pressure at state D. The same pressure in atmospheres is calculated from  $\frac{p_D}{\text{atm}} =$  [enter]. The results are given in Table 1.

**Table I.** Example 1: Results.

	Pressure (atm)	Volume (l)	Temperature (K)
<b>State A</b>	2.00	2.46	300
<b>State B</b>	2.00	3.28	400
<b>State C</b>	1.50	4.37	400
<b>State D</b>	1.13	4.37	300

Second example (Ioannou et al, 2000, p. 166): Magnetic force on a current wire.

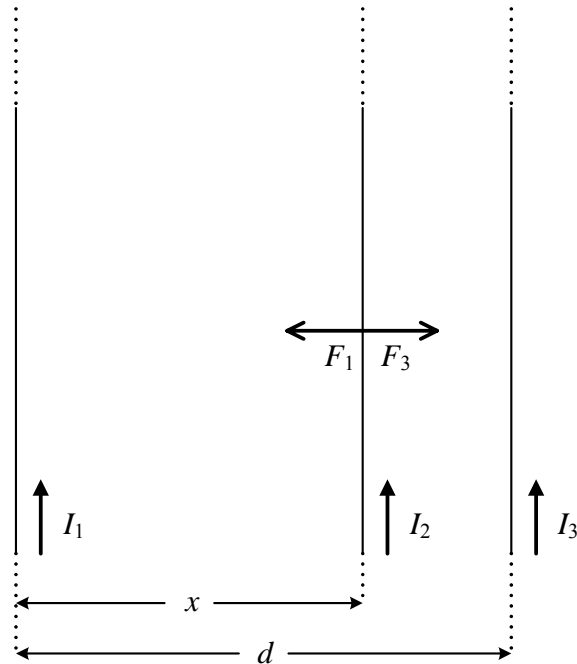
Consider three infinite parallel wires carrying direct current (DC) in the same direction, see picture 1. The wires are arranged in the same plane. The first and the third wire are fixed and separated by a distance  $d$ . Analyze the conditions under which the second wire is steady (force equilibrium).

Obviously, this problem is too complex for a second grade secondary student. In this case, the numerical values of  $I_1$ ,  $I_3$ , and  $d$  are usually given. However, using MathCad® the problem can easily be solved. Suppose that only one current varies at the time. A steady current  $I_0$  flows the other wire (here, we assume that currents vary from 0 to 100A and  $I_0 = 10$ A). Problem is solved as:

$$X(I_1, I_3) := \frac{I_1}{I_1 + I_3} \text{ [enter]}$$

$$I_0 = 10A \text{ [enter]}$$

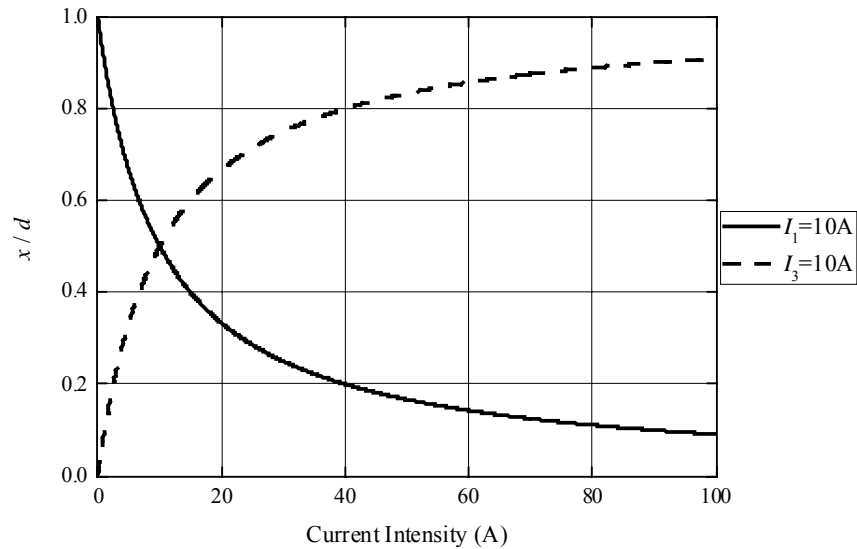
$$I := 0,1A..100A \text{ [enter]}$$



Picture 1. System model.

Then, students select Insert>Graph>X-Y Plot from the toolbar menu which displays on the screen a 2D plot. In the middle placeholder underneath the x-axis (horizontal axis) the range variable  $I$ , and in the middle placeholder underneath the y-axis (vertical axis) the expressions  $X(I_0, I)$  and  $X(I, I_0)$ , are correspondingly entered. Pressing [enter], one point is plotted for each value of the range variable. Students can also save the results to an ASCII file and manipulate them with graphing software. Here, the data have been imported into the Origin<sup>®</sup> software package (<http://www.originlab.com/>). Results are illustrated in Picture 2.

Under the teacher's encouragement and guidance, students can further study and analyze the system. Discussion on issues such as force equilibrium or dependence between cause (current) and result (equilibrium distance) can be included as well. Students may also display 3D plots in a variety of ways (3D bar charts, surface, contours, and 3D scatter plots). In this case, the normalized distance  $x/d$  can be examined when both  $I_1$  and  $I_3$  vary.



**Picture 2.** Normalized distance of the first two wires versus electric current.

Third example (Ioannou et al, 2001, p. 55): Waves propagation.

The waves  $y_1 = 8\sin\left(2\pi\left(\frac{t}{0.3} - 5x\right)\right)$  and  $y_2 = 16\sin\left(2\pi\left(\frac{t}{0.3} + 5x\right)\right)$  propagate in opposite directions in an isotropic elastic medium without energy dissipation (distance is measured in meters and time in seconds). Describe the temporal variation of the total signal at  $x = 3.025\text{m}$ .

The total signal is the sum of the two sinusoids. MathCad® provides students with interest results. The problem is solved as follows (in this example, time varies from 0 to 0.6s):

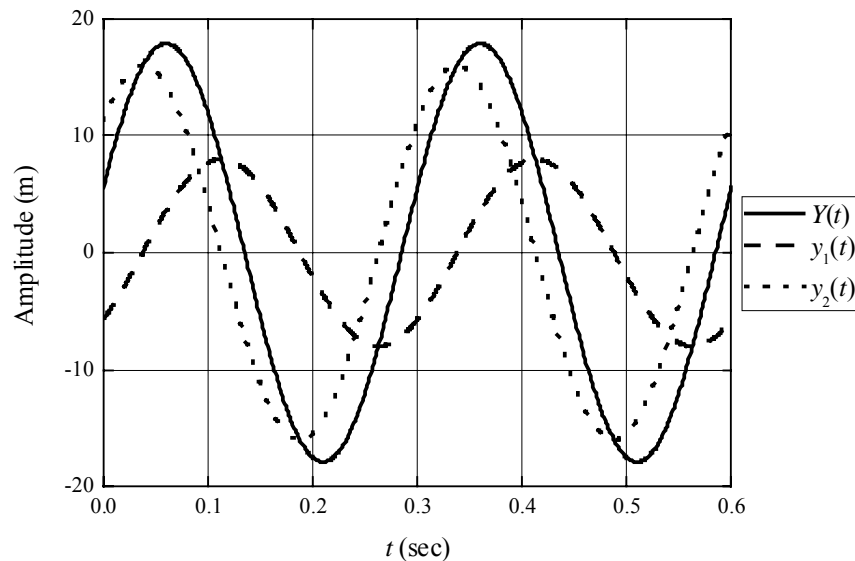
$$y1(x,t) := 8\sin\left[2\pi\left(\frac{t}{0.3\text{s}} - \frac{5}{\text{m}}x\right)\right] \text{m} \text{ [enter]}$$

$$y2(x,t) := 16\sin\left[2\pi\left(\frac{t}{0.3\text{s}} + \frac{5}{\text{m}}x\right)\right] \text{m} \text{ [enter]}$$

$$y(x,t) := y1(x,t) + y2(x,t) \text{ [enter]}$$

$$t := 0, 0.01\text{s}..0.6\text{s} \text{ [enter]}$$

Working as in the previous example, students select Insert>Graph>X-Y Plot from the toolbar menu to display on the screen a 2D plot, and enter the appropriate placeholders, the  $t$  underneath the  $x$ -axis and the  $y(3.025\text{m},t)$ ,  $y1(3.025\text{m},t)$ , and  $y2(3.025\text{m},t)$  underneath the  $y$ -axis. Students can also write the results to an ASCII file and import them into graphing software. Again, the data have been imported into Origin®, see picture 3.



**Picture 3.** Waves amplitudes versus time at  $x=3.025\text{m}$ .

The students can easily calculate the maxima (or minima) and the period of the signals and perform comparisons with theoretical results. Additional characteristics of the propagating signal can also be studied. For example, they can further examine the spatial variation of the signal at a given instant or its dependence on the amplitudes of the initial waves.

It has to be reminded that MathCad<sup>®</sup> is very a powerful mathematical tool with lots of applications in science and engineering. Complicated mathematical operations such as matrix operations, differentiations, integrations, solutions of algebraic systems, etc., give accurate results within a reasonable amount of time. However, these features are out of the scope of the paper, as long secondary students do not entail such sophisticated simulations. Advance educational applications for graduate students can be found in the published literature, (Veglis et al., 2002; Domnisoru, 2005; Li – Khan, 2005; Li – Chaloo, 2006; Sandham – Hamilton, 2006; Dusang – Johnson, 2008) for example.

## 6. Conclusion

In this paper, we have discussed the application of the mathematical software MathCad<sup>®</sup> as a complementary tool in the upper secondary physics education. The package represents a powerful tool for both engineers and students. Specific features make it more suitable in secondary education compared to other commercial packages. These features are the use of mathematical notation similar to its appearance in textbooks and the use of built-in measurement units. Use of the package allows students to focus on results, concepts, and relations to theory and increases their interest in physics. Representative examples have demonstrated its application in the class and exhibited its advantages over traditional teaching approaches. Concluding, MathCad<sup>®</sup> is an interesting proposal for the upper secondary physics education. The proposal was based on the Greek Secondary Education System, but obviously it is applicable in other education systems also.

## Literature

- [1] Andreadis–Papadimitriou, A. et al. (2007), The software of the Greek Ministry of Education for the natural sciences and experiences from its use in secondary education schools, (in Greek), *Proceedings of the 10<sup>th</sup> Congress of Greek and Cypriot Physicists*, B, 9-21, Corfu, Greece.
- [2] Ciascai, L. – Marchis, I. (2008), Multimedia in primary and secondary school curricula in Romania, *Acta Didactica Napocensia*, 1, 2, 76-85.



- [3] Constantinescu, C. G. et al. (2007), Power consumption determination with implications in designing the single-phase transformers working in ac-switching mode, *Proceedings of the IEEE International Symposium on Industrial Electronics ISIE 2007*, 550-554, Vigo, Spain.
- [4] Delgado, J. H. – Thursby M. H. (1999), Implementation of the pyramidal-horn antenna radiation-pattern equations using Mathcad®, *IEEE Antennas and Propagation Magazine*, 41, 5, 96-99.
- [5] Domnisoru, C. (2005), Using MATHCAD in teaching power engineering, *IEEE Transactions on Education*, 48, 1, 157-161.
- [6] Dusang, L. V. – Johnson, B. K. (2008), Evaluation of fault protection methods using ATP and MathCAD, *Proceedings of the 2008 IEEE Electric Power and Energy Conference*, Vancouver, Canada.
- [7] Esa, M. et al. (2006), Design of microwave direct-coupled cavity waveguide filter for VSAT communication at C-Band using MathCAD software, *Proceedings of the IEEE International RF and Microwave Conference RFM2006*, 116-120, Putra Jaya, Malaysia.
- [8] Fuhrman, C. P. (2007), Exploiting open-source projects to study software design, *Informatics in Education*, 6, 1, 53-66.
- [9] Gilat, A. (2008), *MATLAB: An Introduction with Applications*, John Wiley & Sons, Inc.
- [10] Gorman, M. E. et al. (2005), *Scientific and Technological Thinking*, Lawrence Erlbaum Associates, Inc.
- [11] Halalu, W. M. – Elwakil, A. S. (2004), Research-oriented junior/senior design projects: An analog circuit design example, *IEEE Transactions on Education*, 47, 1, 93-99.
- [12] <http://www.ekep.gr/>
- [13] <http://www.originlab.com/>
- [14] <http://www.pi-schools.gr/>
- [15] <http://www.ptc.com/>
- [16] <http://www.ypepth.gr/>
- [17] Ioannou, A. et al. (2000), *Unified Lyceum Class 2 Physics*, (in Greek), Textbook Publishing Organization (O.E.D.B.).
- [18] Ioannou, A. et al. (2001), *Unified Lyceum Class 3 Physics*, (in Greek), Textbook Publishing Organization (O.E.D.B.).
- [19] Li, S. – Chaloo, R. (2006), Restructuring an electric machinery course with an integrative approach and computer-assisted teaching methodology, *IEEE Transactions on Education*, 49, 1, 16-28.
- [20] Li, S. – Khan, A. A. (2005), Applying IT tools to a laboratory course for measurement, analysis, and design of electric and electronic circuits, *IEEE Transactions on Education*, 48, 3, 520-530.
- [21] Kezunovic, M. et al. (2004), The role of digital modeling and simulation in power engineering education, *IEEE Transactions on Education*, 19, 1, 64-72.
- [22] Knobel, R. (2000), *An Introduction to the Mathematical Theory of Waves*, American Mathematical Society.
- [23] Mallick, J. A. – Crawford, M. (2005), Determining pulsed alternator thyristor converter firing angles to produce a desired launcher current, *IEEE Transactions on Magnetics*, 41, 1, 322-325.
- [24] Mamoukaris, K. V. et al. (2000), Educational software for mathematics; a Greek reality, *Proceedings of the International Workshop on Advanced Learning Technologies IWALT 2000*, 79-80, Palmerston North, New Zealand.
- [25] Mathsoft (2001), *Mathcad User's Guide*, Mathsoft, Inc.

- [26] Mathworks (2001), *Using MATLAB*, The Mathworks, Inc.
- [27] Maxfield, B. (2006), *Engineering with Mathcad*, Butterworth-Heinemann.
- [28] Müller, I. (2007), *A History of Thermodynamics*, Springer.
- [29] Neff, H. P. Jr. (1991), *Introductory Electromagnetics*, John Wiley & Sons, Inc.
- [30] Papert, S. (1993), *The Children's Machine: Rethinking School in the Age of the Computer*, Basic Books.
- [31] Piaget, J. (1975), *Equilibration of Cognitive Structures*, University of Chicago Press.
- [32] Pritchard, P. J. (2007), *MathCad: A Tool for Engineering Problem Solving*, 2<sup>nd</sup> ed., McGraw-Hill.
- [33] Roberts, T. S. (2005), *Computer-Supported Collaborative Learning in Higher Education*, Idea Group Publishing.
- [34] Sandham, W. A. – Hamilton, D. J. (2006), Biosignal processing and analysis using Mathcad® - Pedagogical and Research Issues, *Proceedings of the 28<sup>th</sup> IEEE EMBS Annual International Conference*, 6177-6180, New York, USA.
- [35] Szállassy, N. (2008), Project method, as one of the basic methods of environmental education, *Acta Didactica Napocensia*, 1, 2, 44-49.
- [36] Tam, P. T. (2008), *A Physicist's Guide to MATHEMATICA*, 2<sup>nd</sup> ed., Academic Press.
- [37] Vallim, M. B. R. et al. (2006), Practicing engineering in a freshman introductory course, *IEEE Transactions on Education*, 49, 1, 74-79.
- [38] Veglis, A. A. et al. (2006), Teaching performance evaluation of multiprocessor architectures with Mathcad and MathConnex, *IEEE Transactions on Education*, 45, 3, 231-237.
- [39] Vygotsky, L. S. (1978), *Mind in Society: The Development of Higher Psychological Processes*, Harvard University Press.
- [40] Wolfram, S. (2003), *The Mathematica Book*, 5<sup>th</sup> ed., Wolfram Media.

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